How I view 4D as two 2D with texture on there surface plane.

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A 3D object can be represented by the family of planes passing through it or extending from it toward infinity. By intersecting these lines with two reference planes, preferably parallel planes, each line can be encoded by a pair of plane-intersection coordinates. These coordinates provide the continuous geometric information of the plane and are determined by 2 planes differences each.

On top of that continuous line data, each plane also carries a eight-state orientation structure. This comes from four above/below relations: one relation relative to the first reference plane, and another relation relative to the second reference plane. and each plane intersects making a contradiction side that is geometrically allowed Combining these four binary relations gives eight possible states: above/above/above/above, bellow/bellow/bellow/bellow and there combinations. Therefore, the plane is not a single binary object, and it is not merely a two-state object. It is a quad binary object whose combined orientation forms a eight-state structure.

Most lines contain all the information needed to distinguish between these eight cases, while special cases, such as plane parallel to the planes, may collapse two of the distinctions. This eight-state structure creates a discrete layer over the continuous geometry of the plane-plane intersections. the two planes can also intersect making it 4 bits and 16 states but they are defined as parallel but this alternate to parallel gives higher order differences slightly harder to convert to planes after.

If phase-like or texture-like information can be assigned to the differences between these four orientation states, the resulting structure built by reconstructing the planes that make up this, the two binary values of the planes can be given a 90° each and then the other also given 90° each then given each a solution as a sphere though…

to get a sphere you use level sets just flip two voter planes and you get a bloch sphere it is easier to not start from two textured planes but tons of planes instead, you can test it with random planes.

1. THE VOTER PLANE ENSEMBLE

Let $\Omega \subset \mathbb{R}^3$ be a convex polyhedron. A voter plane is:

$$P_{n,\square} = \{ x \in \mathbb{R}^3 : \square \cdot x = \square \}, \quad \square \in \mathbb{S}^2, \ \square \in \mathbb{R}$$

The intersection polygon:

$$\pi(\square,\square) = P_{n,\square} \cap \partial\Omega$$

2. THE 8-STATE OCTANT CLASSIFICATION

The normal $\square = (n_x, n_y, n\square)$ defines a sign vector:

$$\square(\square) = (\operatorname{sgn}(n_x), \operatorname{sgn}(n_y), \operatorname{sgn}(n\square)) \in \{+1,-1\}^3$$

Octant index:

$$\square(\square) = \Sigma_{i=0}^{2} 2^i \cdot \mathbb{1}[n_i < 0] \quad \in \{0,1,\ldots,7\}$$

3. THE FACE SHEET

For a fixed octant □, the face sheet is:

$$\mathcal{F}□ = \bigcup_{□\in\mathbb{S}^2,\ □(□)=□} \pi(□,0)$$

Theorem: For $\Omega = [-1,1]^3$, each $\mathcal{F}□$ traces exactly the three faces visible from direction □, and:

$$\partial\Omega = \bigcup_{□\in\{+1,-1\}^3} \mathcal{F}□$$

4. THE TEXTURE FIELD ON $\mathbb{S}^2$

Each plane carries a texture — a geometric descriptor of π(□,0).
The simplest texture is area:

$$□(□) = \text{Area}(\ \pi(□,0)\ )$$

This defines a scalar field:

$$□ : \mathbb{S}^2 \to \mathbb{R}_+$$

For the cube:

$$□(□) = 4 \qquad \text{at the six cardinal directions (square)}$$
$$□(□) = 3\sqrt{3} \approx 5.2 \qquad \text{at the body diagonals (hexagon)}$$

5. THE TWO-PLANE PARAMETERIZATION

Two parallel reference planes Π_1, Π_2 separated by distance h.
A line □ is parameterized by piercing points:

$$□ \mapsto (□_1, □_2) \in \Pi_1 \times \Pi_2 \cong \mathbb{R}^2 \times \mathbb{R}^2$$

The 4 binary relations (above/below per plane):

$$□_1 = \mathbb{1}[\ □_1 \text{ is above } \Omega \text{ on } \Pi_1\ ]$$
$$□_1 = \mathbb{1}[\ □_1 \text{ is below } \Omega \text{ on } \Pi_1\ ]$$
$$□_2 = \mathbb{1}[\ □_2 \text{ is above } \Omega \text{ on } \Pi_2\ ]$$
$$□_2 = \mathbb{1}[\ □_2 \text{ is below } \Omega \text{ on } \Pi_2\ ]$$

Combined: $2^4 = 16$ states. Parallel planes reduce to 8 effective states.
The contradiction side is the line orientation relative to the object center.

6. THE BLOCH SPHERE CLOSURE

Given two texture fields $□_\alpha$, $□_\beta$ on $\mathbb{S}^2$, define the difference cylinder:

$$□_{\alpha\beta} = \{\ (□,\ □,\ □)\ :\ □ = □_\alpha(□,□) - □_\beta(□,□)\ \} \subset \mathbb{S}^1 \times □$$

The closure operation assigns a 90° phase to each binary distinction and resolves the interference into a sphere:

$$|□\rangle = \Sigma_{□\in\{+1,-1\}^3}\ \sqrt{□□}\ \cdot\ e^{\{i□□\}}\ |□\rangle$$

The Bloch sphere is the projective space:

$$\mathbb{S}^2 = \mathbb{CP}^1 \cong □(□\mathcal{H}_8)$$

where $\mathcal{H}_8$ is the 8-dimensional Hilbert space spanned by the octant basis.

7. THE RECONSTRUCTION ALGORITHM (RANDOM PLANE VOTING)

Input: Object Ω, number of voters N
Output: Point cloud reconstruction of ∂Ω

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for i = 1 to N:
    Sample □ᵢ uniformly from 𝕊²
    Compute □ᵢ = sign(□ᵢ)                    // 8-state
    Compute πᵢ = P_{□ᵢ,0} ∩ ∂Ω               // intersection polygon
    for each vertex □ ∈ πᵢ:
        emit point (□, □ᵢ) to cloud
return ⋃ᵢ { □ : (□, □ᵢ) }
```

No integration. No density estimation. No silhouette logic.
Pure incidence geometry.